QP CODE: 21100830

21100830

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) EXAMINATION, MARCH 2021

Fourth Semester

Complemetary Course - MM4CMT01 - MATHEMATICS - FOURIER SERIES, LAPLACE TRANSFORM AND COMPLEX ANALYSIS

(Common for B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology Model I,B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Chemistry Model III Petrochemicals, B.Sc Physics Model III Electronic Equipment Maintenance, B.Sc Geology and Water Management Model III)

2017 Admission onwards

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Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define Fourier Cosine Series?
2. Write the Legendre Polynomials $P_0(x)$ to $P_5(x)$
3. Find $\mathscr{L}(0)$
4. Write $\mathscr{L}(\int_0^t f(t)dt)$ in terms of $\mathscr{L}(f(t))$
5. Write $\mathscr{L}\{t^2 f(t)\}$ using a derivative of $\mathscr{L}\{f(t)\}$
6. Find the real and imaginary parts of $z_1 z_2$ where $z_1 = 8 - 3i$ and $z_2 = 9 + 2i$.
7. Find the real and imaginary parts of $\frac{1}{z}$ where $z = 2 - 3i$.
8. Define the derivative of a complex function f at a point z_0.
9. Find $|e^z|$, where $z = 2 + 3\pi i$.
10. Define simple closed contour.
11. State Cauchy's integral formula.
12. State Liouville's Theorem.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the fourier series expansio of
$f(x) = \frac{\pi - x}{2}, 0 < x < 2\pi, f(x + 2\pi) = f(x)$ and deduce that
$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$

14. Expand $f(x) = \pi x - x^2, x \in [0, \pi]$ in fourier cosine series?

15. Evaluate $\mathcal{L}^{-1}\{\frac{s+a}{s^2(s+a)}\}$

16. Solve $y'' + 4y = 0$ given $y(0) = 2$ and $y'(0) = -8$, applying Laplace transforms.

17. Show that $|z_1 + z_2| \leq 3$, where $z_1 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $z_2 = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$.

18. Find all the roots of $\sqrt[3]{1+i}$.

19. Find the real and imaginary parts of $cosh(-3 - 6i)$.

20. Find an upper bound for the absolute value of the integral $\int_C z^2 dz$, where C is the straight line segment from 0 to 3+ 4i.

21. Evaluate $\oint_C \frac{e^z}{(z-1)^2(z^2+4)} dz$, C is the circle |z| = 3/2.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Solve using power series method
    a) $(1 + x)y' = y$
    b) $y'' = 4y$

23. Find (a) $\mathcal{L}^{-1}(\frac{4s+1}{s^2-16})$

    (b) $\mathcal{L}^{-1}\{\frac{4}{(s+1)(s+2)}\}$

24. Check whether $v = -e^{-x} siny$ is harmonic or not. If YES, find a corresponding analytic function.

25. Show that $\oint_C \frac{1}{z} dz = 2\pi i$ where C is the unit circle with centre origin. Does this contradict Cauchy's integral theorem? Justify your answer.

(2×15=30)